

MATH REVIEW: HW II

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1. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$.
2. Show that if the x- and y-intercepts of a line are nonzero numbers a and b , then the equation of the line can be put in the form:

$$\frac{x}{a} + \frac{y}{b} = 1$$

3. Describe in words the region of \mathbb{R}^3 represented by the equation or inequality:

a) $y = z$

b) $x^2 + z^2 \leq 9$

c) $1 \leq x^2 + y^2 + z^2 \leq 25$

d) $xyz = 0$

4. Prove that every tangent plane to $z = xf(y, x)$ passes by $(0, 0, 0)$.
5. Graph the function, and find all its maxima and minima (either global or local):

a) $\Gamma(x) = \begin{cases} 1 - x, & \text{if } 0 \leq x < 2 \\ 2x - 4, & \text{if } 2 \leq x \leq 3 \end{cases}$

b) $\Psi(x) = \begin{cases} 2 - x^2, & \text{if } -2 \leq x < 0 \\ 2x - 1, & \text{if } 0 \leq x \leq 2 \end{cases}$

6. Sketch the graph:

a) $f(x) = e^x/x$

b) $g(x) = 8(x^3 - x)$

7. (*) Let $\varphi : \mathbb{R} \rightarrow \mathbb{R}$.

a) Find a general expression for the line L that passes through φ at $(x_1, y_1), (x_2, y_2)$.

b) Prove that φ is strictly increasing, if $\forall x_2 > x_1; x_1, x_2 \in \text{Dom}(\varphi)$ the slope of L is positive.